Home work for the atelier *Modélisation*.
To be finished before 17h30, Thursday 15 november 2012.

Logistic population dynamics & harvesting

Fixed quotas *vs* proportional quotas

Which of the following two models of harvesting predicts a tipping point? The objective is to analyse the two models, and to identify which model has a catastrophic transition related to a fold bifurcation when the harvesting rate H is gradually increased.

The basic model: Logistic growth.

One of the simplest and most-used ecological models of population dynamics is the logistic model:

$dN/dt = rN\,(1-N/K)$

where N is the population size (say biomass), r is the "instrinsic growth rate" (exponential growth rate at low N), and K is the carrying capacity.

1. Analyse the model as follows: sketch dN/dt as a function of N. Identify the equilibria and their stability.
2. Analyse the two models below in similar way.
3. Try to sketch the bifurcation diagrams for the two models: sketch the equilibrium (of N) as a function of H, for all equilibria present, and indicate stability of the different equilibria.
4. Answer to question posed above: Which of the following two models of harvesting predicts a tipping point?

Harvesting model 1: fixed quotas

One way to model harvesting is to assume that a fixed quantity is harvested per unit of time. This mimicks (in a very much simplified way) the most common fisheries policy called "fixed quotas".

$dN/dt = rN\,(1-N/K) - H$

where H is the harvesting rate (biomass taken per unit of time).

Harvesting model 2: proportional quotas

Another popular way to model harvesting is to assume that each individual (or unit of biomass) is exposed to a fixed harvesting rate. The total harvesting rate is then proportional to the current population size N.

$dN/dt = rN\,(1-N/K) - H\,N$